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A RANDOMIZATION TEST FOR COMPARING 1/4-SCALE KINETIC ENERGY PENETRATORS

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Material properties of kinetic energy penetrators are compared at the Ballistic Research Laboratory in a 1/4-scale test environment. Metallurgists fire penetrators of various material compositions into semi-infinite steel blocks and record depths of penetration. Depth of penetration behaves approximately as a linear function of velocity, d(v), over the range of the four-velocity design routinely employed. Under a common slopes assumption, a difference in performance between penetrators *k* and *l* is computed as $d_k(v) - d_l(v)$. This difference is determined graphically, occasionally with the benefit of a least-squares fit to each performance. Statements of significance are not made at present. The approach presented is an attempt at a unifying structure within which inferences in this environment can be made both quantifiable and consistent. The recommended procedure combines existing techniques such as least squares with a new application of a randomization test in determining the significance of observed material differences. With this test supporting, practitioners can make definitive statements as to the statistical significance of material differences observed.

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1. INTRODUCTION

Material properties of kinetic energy penetrators are compared at the Ballistic Research Laboratory in a 1/4-scale test environment. Metallurgists fire penetrators of various material compositions into semi-infinite steel blocks and record depths of penetration. Depth of penetration behaves approximately as a linear function of velocity, d(v), over the range of the four-velocity design routinely employed. Under a common slopes assumption, a difference in performance between penetrators k and l is computed as $d_k(v) - d_l(v)$. This difference is determined graphically, occasionally with the benefit of a least-squares fit to each performance. Statements of significance are not made at present. In this paper, a randomization test is presented as a means for providing analytical support for inference.

Inferences drawn from such experimentation may be considered the result of meta-analysis. Meta-analysis is loosely described as the "integration of independent studies" by Hedges and Olkin (1985). This area has received much recent attention in the social and biological sciences, but in the physical and engineering sciences it has received little notice with the exception of a few historical papers (e.g., Tippet [1931] and Fisher [1932]) that have been classified in retrospect as meta-analyses. The independent-studies quality of the aforementioned problem stems from the combination of data sets gathered at different times (often different years) and by different experimenters. This fact, practically speaking, invalidates a necessary assumption for normal theory analyses, namely the belief that the subjects for the combined data set are the result of a random sample.* Taylor and Bodt (1991) recommend surmounting this problem through the use of randomization tests and demonstrate applicability of this methodology to significance testing with ballistic data.

The purpose of this paper is to introduce a randomization test for comparing 1/4-scale kinetic energy penetrators. A description of the data collection is followed by the discussion of a linear model through which significance testing of relevant contrasts can be made. It is then demonstrated how a reference distribution for determining significance can be achieved through randomization. Application of the procedure and discussion of the results follow.

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* In an ideal situation one would design a multiyear experiment where random sampling did occur, but the obstacles are so formidable in this testing environment that it is not done.

2. THE DATA COLLECTION

The measured response, d_{ij}, is the depth of penetration permitted by a semi-infinite steel block subjected to a hemi-nose penetrator of material j, fired at velocity i. Semi-infinite describes the independence of the penetration action to influences from side and rear free surfaces (i.e., the block is for practical purposes infinite with respect to width and depth). Hemi-nose refers to the hemispherical configuration of the projectile nose. Figure 1 shows the cut-away profile of a semi-infinite block, where the cut is made along the shot line. Depth of penetration is taken to be the maximum normal distance between the original entry-point surface and the bottom surface of the hole.

Depth of penetration from penetrators of several different material compositions are gathered over several velocities. The design structure suggests that the experimental units are the semi-infinite steel blocks. It is these that are exposed to the two treatments, velocity and penetrator material. Velocity is included as a test condition because it will affect penetration depth. Penetrator material is the only treatment of interest – materials are to be compared for relative effectiveness. Confidence in the assessment of relative performance is ensured through comparison over a range of velocities meaningful to the Army application (i.e., over a typical ordnance velocity range). A template for the experiment is to fire each penetrator (material) once at each of the following four nominal velocities: 1100 m/s, 1300 m/s, 1500 m/s, and 1700 m/s. Actual velocities will vary. A design matrix overlaid on a combined data set including different materials might appear as Figure 2.

Other facets of data collection influence the analysis. Penetrators are tested in separate experiments, quite possibly over as many as ten years if the purpose is to compare new materials to an historical control. Small sample sizes with no replication prevail if one adheres to the template for testing materials. There is no random sampling from a population of semi-infinite blocks – indeed, at the time of the first experiment, blocks used in later firings may have not yet been manufactured. Even if the sample were random, there is no guarantee that the population is normal, nor is it likely that the comfort of approximate normality can be afforded by the Central Limit Theorem with the sample sizes and replication considered.

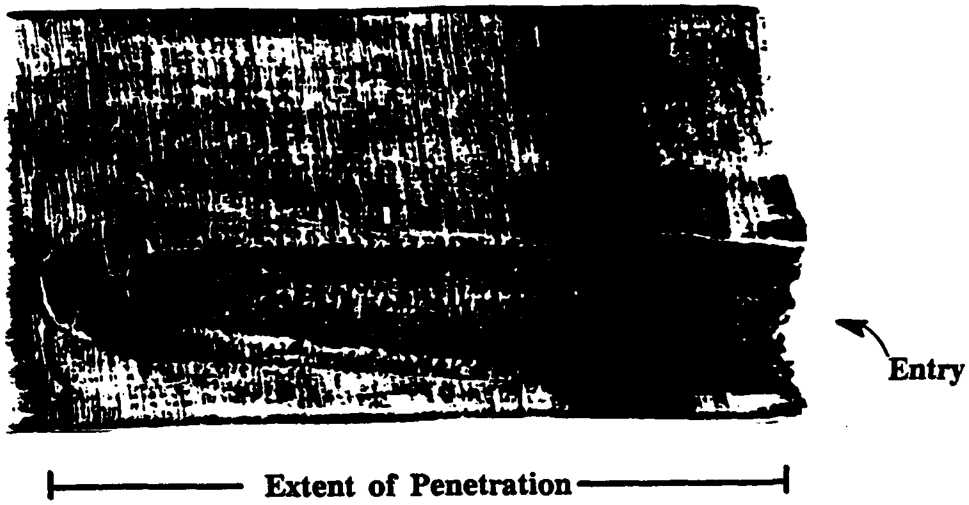

Figure 1. Cut-Away Profile of a Semi-Infinite Steel Block After Penetration.

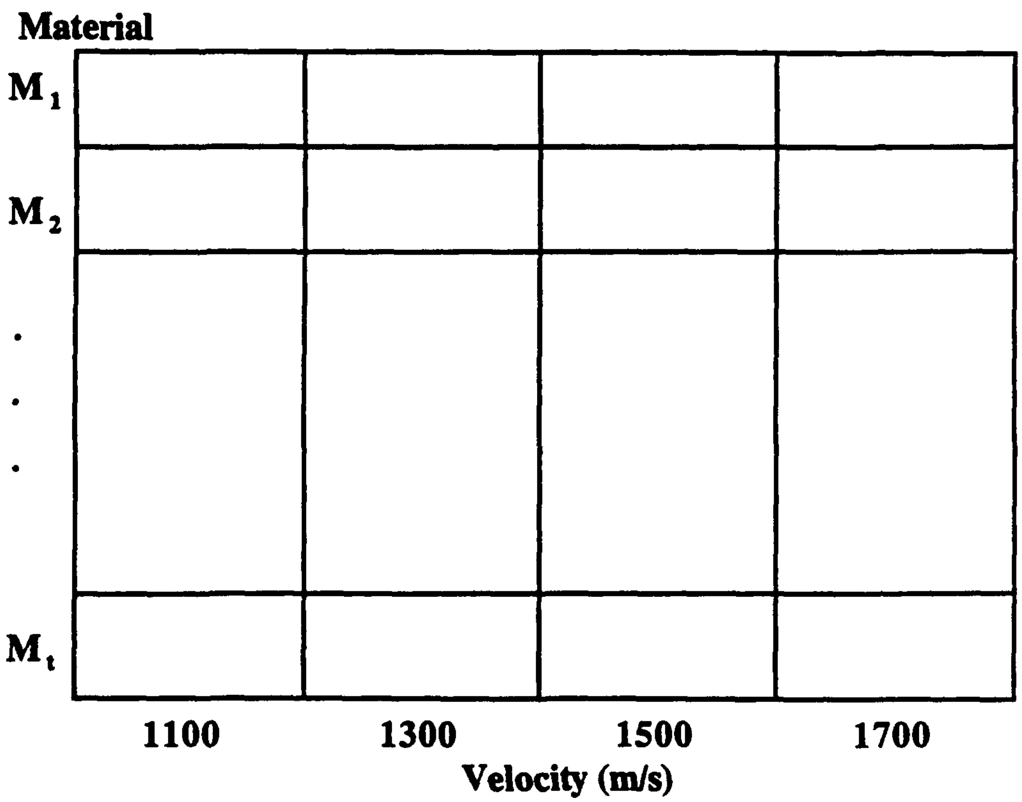

Figure 2. Template for Data Collection.

3. THE LINEAR MODEL

A linear models framework is presented in this section to support inference for this problem. Great detail is not given. For a comprehensive, but introductory, treatment, it is suggested the reader turn to Neter and Wasserman (1974). The problem is first described in the context of a two-factor factorial design, followed by a refinement in the form of an analysis of covariance model. A convenient regression form of this model is then used to construct meaningful contrasts. Assumptions required for traditional significance testing of those contrasts are also discussed.

3.1 Factorial Design. The design matrix shown in Figure 2 and the problem description suggest that a factorial design may be appropriate, with penetrator material serving as the principal treatment under study and velocity serving as an additional design variable. The additive model is expressed

$$d_{ij} = \mu + V_i + M_j + e_{ij}, \tag{1}$$

where μ is the common mean response, V_i and M_j are the effects (shifts from that mean) caused by the i^{th} velocity and the j^{th} material, respectively, and e_{ij} is the error associated with the $(ij)^{th}$ response. We begin by assuming a Model-I stance, indicating that both material and velocity be treated as fixed effects.

Two facts render this approach less than ideal. The first, stated in the Introduction, is that experimenters know that velocity behaves approximately linearly with penetration depth. Even further, experience has shown that $d_k(v)$ and $d_l(v)$ are virtually parallel over the 1100 m/s to 1700 m/s velocity regime, hence the additivity assumption above. Beyond this regime the assumptions of linearity and parallel lines do not hold.* The second is that although four nominal velocities are intended, the actual velocities tested often number as many as the number of 1/4-scale rods fired. Because firing velocity cannot be completely controlled, each nominal velocity actually encompasses a range of velocities close to the nominal. Figure 3 illustrates both linearity and firing velocity noise in replication of tungsten alloy firings at the four nominal velocities.

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* Current engineering thought, supported by high velocity testing results, is that the lines begin slowly converging over this velocity regime, with more rapid convergence occurring well beyond 1700 m/s.

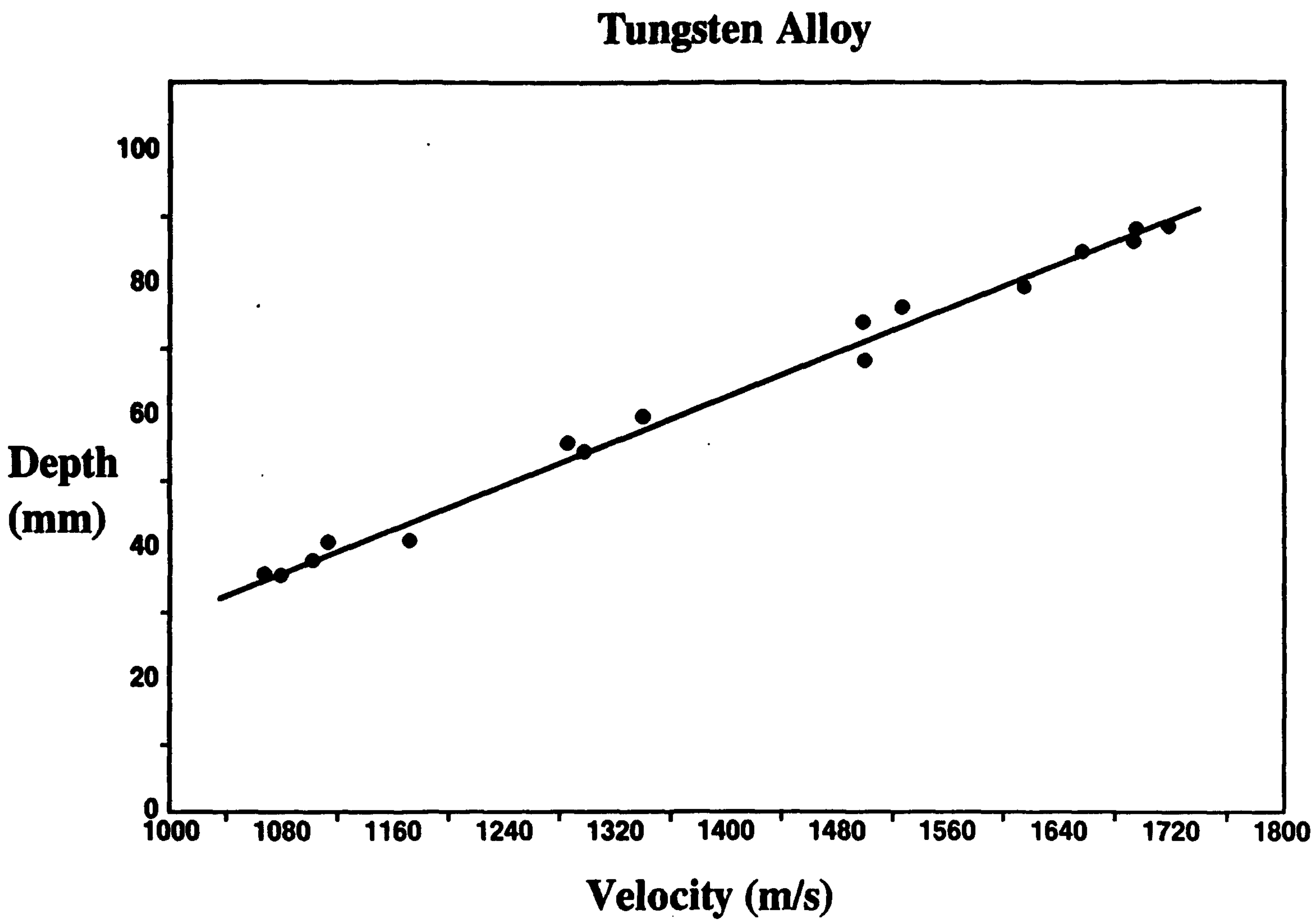

Figure 3. Tungsten Alloy Depth of Penetration Plotted Against Velocity.

This additional information impacts the method of analysis. Taking advantage of linearity would save the experimenter degrees of freedom to apply in the estimation of error – more efficiency in the model is possible. Left unconsidered, firing velocity noise would increase the estimate of response variability. In the next section the analysis of covariance model is suggested, having the advantage that both linearity and firing velocity variation can be incorporated.

3.2 Analysis of Covariance.

3.2.1 Traditional Model. The linear relationship between velocity and depth of penetration can be made part of the linear model as follows. Rewrite Equation 1 as

$$d_{ij} = \mu + (\mu_{i.} - \mu) + (\mu_{.j} - \mu) + (d_{ij} - \mu_{i.} - \mu_{.j} + \mu), \tag{2}$$

where μ is again the common mean response and $\mu_{i.}$ and $\mu_{.j}$ are the true mean depths of penetration associated with the i^{th} velocity and j^{th} material, respectively. Replace the mean-shift interpretation for the velocity effect $\mu_{i.} - \mu$ with $\mu_{d/v}$ to represent the simple linear relationship between velocity and the mean response. Adding and subtracting $\mu_{d/v}$ in the right side of Equation 2 and rearranging terms leaves

$$d_{ij} = \mu_{d/v} + (\mu_{.j} - \mu) + (d_{ij} - \mu_{d/v} - \mu_{.j} + \mu). \tag{3}$$

Let v_{ij} represent the velocity of the $(ij)^{th}$ penetrator, where the index i need not reflect the nominal velocities in Figure 2. The simple linear model which regresses penetration depth on velocity can then be expressed as $\mu + \gamma(v_{ij} - \overline{v}_{..})$, where γ is the slope of the regression and $\overline{v}_{..}$ is the sample mean velocity based on velocity observations taken over both indices. Substituting this for $\mu_{d/v}$ in Equation 3 yields

$$d_{ij} = \mu + \gamma(v_{ij} - \overline{v}_{..}) + (\mu_{.j} - \mu) + (d_{ij} - \gamma(v_{ij} - \overline{v}_{..}) - \mu_{.j}), \tag{4}$$

which the reader recognizes as the common form of an analysis of covariance model.

Certainly, the analysis of covariance model in Equation 4 has appeal in that it can account for the contribution to penetration depth from individual velocities; whereas, in the factorial design the contribution of nominal velocities are counted as being the same regardless of noise. Further, even if the nominal velocities were exactly achieved, there is advantage to be gained in introducing the linearity information in the model. In that case, degrees of freedom for estimating error are saved. The factorial design allows s - 2 fewer degrees of freedom for error, where s represents the number of nominal velocities. This follows directly from the fact that the factorial design requires s - 1 degrees of freedom be assigned to velocity; whereas, the simple linear regression needs only one degree of freedom assigned to the slope to account for the influence of velocity. If the regression is perfect (i.e., fits exactly to the mean response for each nominal velocity), the sum of squares associated with error for both models is identical, leaving analysis of covariance with a decided advantage. If the regression is not perfect, a tradeoff is made wherein degrees of freedom for the error term denominator are gained at the expense of the regression lack-of-fit being added in the numerator. In consideration of data with a strong linear relationship like those displayed in Figure 3, an analysis of covariance approach would be a more appropriate choice than the two-factor factorial.

Using the analysis of covariance model to describe the problem structure, questions regarding material comparisons can be answered through the study of contrasts. If the experimenter is interested in the difference in the effect of any two materials k and l, the contrast $M_k - M_l$ (i.e., $\mu_{.k} - \mu_{.l}$) would be estimated and then tested for significance.

3.2.2 Regression Formulation. It is convenient to reformulate Equation 4 in terms of a regression model. From an applications perspective, the least-squares approach is more widely understood and accepted by practitioners.* Moreover, the parameters have greater intuitive appeal, and their meaning conforms to how experimenters at the Ballistic Research Laboratory currently think of the problem.

The change is accomplished easily. Replace the material effect, having t distinct levels, with indicator variables m_{ijk}, $k = 1,2, \cdots, t-1$, defined such that

$$m_{ijk} = 1 \text{ if the observation is of material k;}$$
$$= 0 \text{ otherwise.}$$

The columns in the regression design matrix corresponding to the indicator variables

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* Regression is also of use, computationally, when the design matrix is unbalanced.

will be mutually orthogonal. Thus, Equation 4 may be expressed in terms of a regression model as

$$d_{ij} = \beta_0 + \beta_1 m_{ij1} + \beta_2 m_{ij2} + \cdots + \beta_{t-1} m_{ij,t-1} + \beta_t(v_{ij} - \overline{v}_{..}) + e_{ij}, \text{ where} \tag{5}$$

$$\beta_0 = \mu + M_t,$$
$$\beta_k = M_k - M_t, \quad k = 1,2, \cdots t\text{-}1,$$
$$\beta_t = \gamma.$$

The coefficients β_k, $k = 1, 2, \cdots, t-1$ represent the difference between the effect of the k^{th} and t^{th} material (i.e., the vertical difference between the parallel regression lines $d_k(v) - d_t(v)$). The designation of the t^{th} material is arbitrary, determined by how the indicator variables are defined. In the design matrix for the regression model, the rows corresponding to the t^{th} material would have zeros in the columns corresponding to the $t-1$ indicator variables. The interpretation of the β_k's would be most natural if a reference group or an historical control was denoted the t^{th} material. Other comparisons may also be of interest. The general contrast $M_k - M_l$, $k, l \neq t$ is obtained through the difference $\beta_k - \beta_l$.

In this section the treatment effects were expressed in the context of a regression formulation of the analysis of covariance model. Estimation of these effects can be accomplished after first determining the least squares estimate of the coefficient vector. The next step – and the main focus of this effort – is to determine the significance of these effects. To begin, we consider conditions for test validity.

3.2.3 Assumptions. Several assumptions are required to support the usual analysis of covariance for this problem. They appear as follows: 1) the regression slopes are nonzero and homogeneous among materials, 2) velocity is unaffected by material, 3) velocity is precisely measured, 4) model errors are distributed with zero mean and common variance, and 5) the responses are considered jointly independent normal random variables. The practical implication of 4) and 5) together is that penetration depths to be allowed by the semi-infinite blocks constitute a random sample from some conceptual normal population.

The first four assumptions are accepted; the last is not. Velocity obviously affects penetration depth, and data support the similar-slopes claim. All test penetrators are identical in geometry; there is no reason to expect that velocity will be influenced by which material composition is being tested. Velocity, though not completely controlled, is precisely measured using an x-ray multiflash system. The fourth assumption is common to nearly all modeling efforts. Replicate data provide a basis for investigating the common variance claim, but error with zero mean must remain without check. As for the last assumption, there is no reason to expect that penetration depths are normal, and because of the individual-study nature of the experiments, they do not constitute a random sample.

In Section 4 we relax this last assumption to require only that the penetration depths be pairwise uncorrelated. With that change, the least-squares estimation of the parameters in Equation 5 will retain the usual properties of uniform minimum variance among linear unbiased estimators but without any known distribution on which to base tests of significance. Under these revised model assumptions, an alternative test for significance is given.

4. A RANDOMIZATION TEST

In this section a randomization test is proposed as a means to discern among statistically different materials. Its principal advantages are freedom from the assumption that data under consideration constitute a random sample from a normal population and the ability to provide exact significance levels. Some basic foundation is followed by a description of the test.

4.1 Foundation. A randomization test is a method through which significance testing is accomplished, with the sampling distribution of the test statistic derived from permutations (combinations) of the data. A test of significance measures the numerical evidence against a conjecture. Data, conveyed through a suitable test statistic, are examined as to their consistency with the conjecture by comparing the observed value of the test statistic to its sampling distribution – formed assuming the conjecture is true. Degrees of inconsistency are reflected in how unusual the observed test statistic appears. This appearance is measured in terms of the p-value, the probability that a value of the test statistic is at least as unusual (large or small) as the one observed.

A classical analysis in this 1/4-scale penetrator environment, based on the model of Section 3.2.2, suggests that a conjecture (null hypothesis) of either $H_0: \beta_k = 0$ or $H_0: \beta_k - \beta_l = 0$ might be tested to compare two materials. Consider the latter hypothesis, a claimed equivalence between materials k and l. Letting b denote a least-squares estimate for β, $b_k - b_l$ is the estimated difference between materials k and l (i.e., the estimated vertical distance between their parallel regression lines). To determine whether the distance is statistically significant, one need only compare $b_k - b_l$ to its sampling distribution. This distribution is readily attainable, but only if one is willing to assume a normal random sample – not satisfied here.

Useful significance tests are possible without benefit of assumption 5). In what follows, this assumption is replaced with the less restrictive condition that penetration depths be pairwise uncorrelated, thus guaranteeing desirable properties for the least-squares estimators. Before proceeding we should note that others have circumvented the normality requirement. Nonparametric approaches to this problem include papers by Quade (1967), Puri and Sen (1969), Shirley (1981), Conover and Iman (1982), and Stephenson and Jacobson (1988). All focus on the rank transforms of either the response variable, the concomitant variables, or both. For example, Conover and Iman (1982) transform both sets of variables to ranks and then conduct a parametric analysis of covariance, eventually relying on the F-distribution to determine significance. An exception to complete reliance on ranks is found in Puri and Sen (1969). In that paper general scores, including ranks, are adjusted for regression on the concomitant variables, and the asymptotic distribution of the test statistic based on those scores is developed using permutation. The hypothesis tested is that no difference exists overall among the treatments (materials) studied. A related approach is now described, focusing on the pairwise comparison of materials.

4.2 Description. Consider first $H_0 : \beta_k = 0$.* The geometrical interpretation of β_k is that it is the vertical distance between the parallel regression lines $d_k(v)$ and $d_t(v)$. This fact is evident from Equation 5. The linear effect of velocity can be removed by adjusting the penetration depth values for the velocities used to achieve them – the remaining difference among the adjusted values, excluding random variation, is attributable to material and is expressed β_k. This difference is estimated as b_k by subtracting the average of the residuals resulting from material t from those of material k, the residuals being computed relative to $d_t(v)$ in each case. Thus, once the two groups of residuals are formed, we are interested in the difference in location between them.

* Specifically, the null hypothesis for the randomization test is that penetration depth measurements are stochastically independent of the penetrator having been formed from material k or material t (Edgington 1987).

To determine if this difference is significant, we need only establish a reference distribution and compare the observed difference to it. Under the null hypothesis, $d_k(v)$ and $d_t(v)$ are coincident. Thus, the residuals computed after adjusting for the linear effect of velocity should be homogeneous. Therefore, in computing b_k, the distinction of which residuals resulted from assignment (association) with material k or material t should make little difference. The reference distribution is constructed by computing b_k under all possible assignments of residuals (effectively ignoring material distinction) to the two materials, the cardinality of each material set being preserved.* For example, if material k had five data values and material t had four, there would be ${}_{(5+4)}C_5$ values computed for b_k. The p-value for the two-sided alternative hypothesis is simply the ratio of the number of values in the reference distribution which equal or exceed in absolute value the observed $|b_k|$ to the total number of combinations, ${}_{(5+4)}C_5$.

Significance testing for the hypothesis $H_0: \beta_k - \beta_l = 0$ is achieved similarly. Adjust penetration values for the linear effect of velocity and compute residuals in the same manner, still computing the residuals relative to $d_t(v)$. The difference between materials is estimated by $b_k - b_l$ and computed by subtracting the average of the residuals resulting from material l from those of material k. The reference distribution arises from computing $b_k - b_l$ under all possible assignments of residuals between materials k and l.

Before turning to examples, some more detail is required as to how these residuals, relative to $d_t(v)$ are computed. From Equation 5, the model $d_t(v)$ can be expressed

$$d_t(v) = \beta_0 + \beta_t(v - \overline{v}_{..}). \tag{6}$$

(The indices ij have been suppressed to emphasize that this is a model for penetration depth.) Both β_0 and β_t must be estimated. Begin with slope. Assuming parallel penetration-against-velocity models d(v), the common slope is taken as the average within-materials regression slope, b_t, which can be delivered by any regression

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* This rationale presupposes random allocation of subjects to treatments. However, as pointed out by Edgington (1987), random allocation principally guards against undue influence resulting from between or within subject variability. Such variability in the context of semi-infinite steel blocks is considered negligible relative to the material differences under study.

subroutine fitting the regression expressed as Equation 5 in its complete form. The estimate is computed as

$$b_t = \frac{\sum_i\sum_j (v_{ij} - \overline{v}_{.j})(d_{ij} - \overline{d}_{.j})}{\sum_i\sum_j (v_{ij} - \overline{v}_{.j})^2}.$$

Usually, $\overline{d}_{.t}$ would serve as the estimate for β_0 in Equation 6. However, in an analysis of covariance $\overline{d}_{.t}$ must be adjusted (adj.) for the common slope, leaving

$$\overline{d}_{.t(adj.)} = \overline{d}_{.t} - b_t(\overline{v}_{.t} - \overline{v}_{..}).$$

This too will be delivered by a regression of Equation 5 when zeros are used as the values for the t-1 indicator variables in the data rows corresponding to the t^{th} material. Estimating β_0 and β_t by $\overline{d}_{.t(adj.)}$ and b_t, respectively, the model $d_t(v)$ shown in Equation 6 can be estimated by

$$\hat{d}_{ij(t)} = \overline{d}_{.t(adj.)} + b_t(v_{ij} - \overline{v}_{..}). \tag{7}$$

Equation 7 is merely the least-squares fit for the t^{th} material, taking into consideration the common slope over all materials. The residuals for the j^{th} material relative to the t^{th} material $r_{ij(t)}$ are computed as

$$r_{ij(t)} = d_{ij} - \hat{d}_{ij(t)}.$$

The residuals $r_{ij(t)}$ are then manipulated in the manner described above.

5. EXAMPLES

In this section two examples are discussed. The purpose of the first is to provide a synopsis of how the randomization test is performed. In that example, data are characteristically sparse. The purpose of the second is to illustrate performance when data are slightly more abundant and when the data collection does not exactly follow the template discussed earlier. Data for both examples were extracted from an unpublished manuscript provided by Mr. Timothy Farrand of the Ballistic Research Laboratory.

5.1 Example 1. Figures 4-5 display data arising from the firing of four penetrator (material) types against semi-infinite steel blocks. All penetrators were manufactured

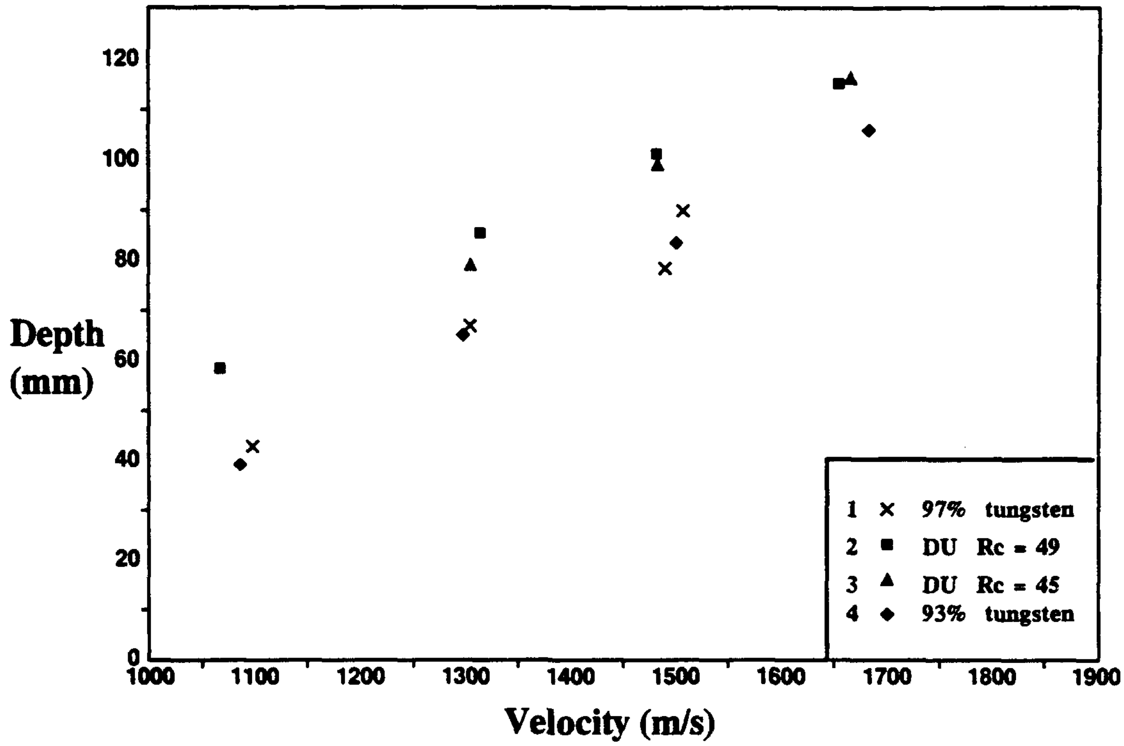

Figure 4. Depth of Penetration for Four Materials, L/D = 15.

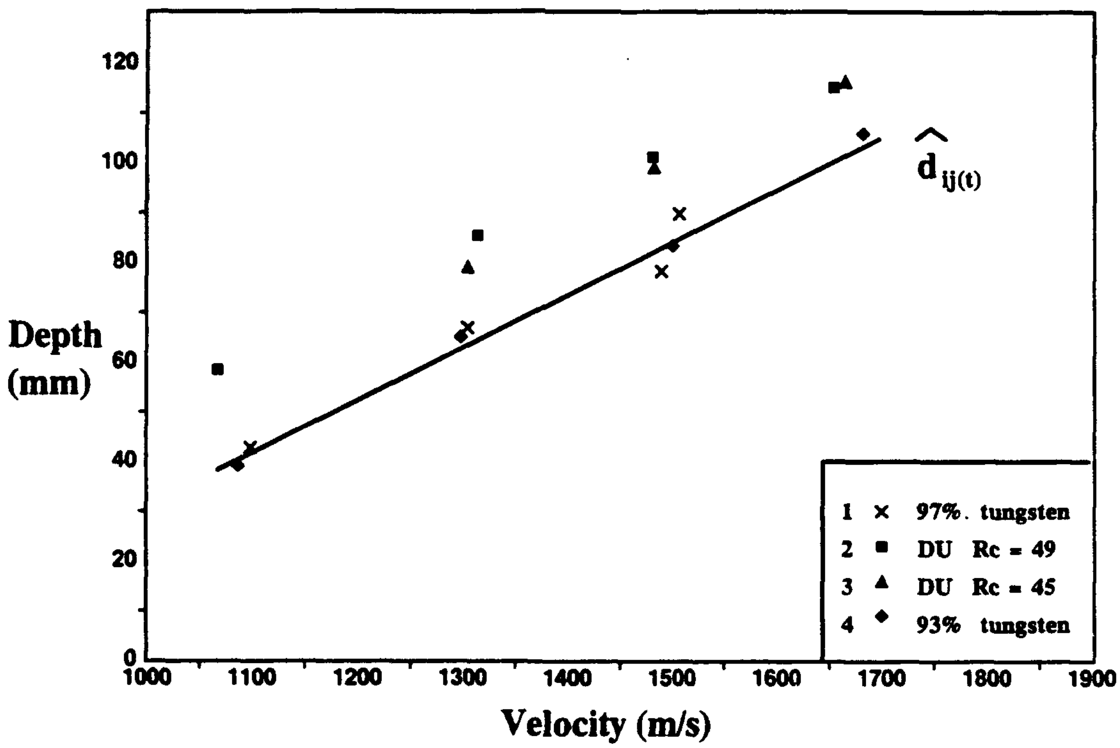

Figure 5. The Relationship of Penetration Depth to the Fit $\hat{d}_{ij(t)}$.

with a common geometry, mass of 65 g, and length-over-diameter ratio (L/D) equal to 15. The depleted uranium (DU) penetrators are separated according to Rockwell hardness (Rc). The template for data collection given in Figure 2 was followed with regard to target velocities. Deviations from the template include duplicate 97%-tungsten results at 1500 m/s and no result for Du Rc=45 at 1100 m/s. Four data points are the most recorded for any material. Data are listed in Table 1.

Two tasks must be accomplished on the way to significance testing. The first step is the estimation of $d_t(v)$. In this example, material t is 93% tungsten. Estimates for the parameters β_0 and β_t will result from regressing penetration depth on velocity and the three indicator variables found in Equation 5. The values for the indicator variables m_{ij1}, m_{ij2}, and m_{ij3} are shown in Table 1. It follows that β_1, β_2, and β_3 represent differences from 93% tungsten (our control) and 97% tungsten, DU Rc=49, and DU Rc=45, respectively. The estimated penetration depths for material t are given by

$$\hat{d}_{ij(t)} = 73.7310 + 0.1035(v_{ij} - 1395)$$

which is graphed as $\hat{d}_{ij(t)}$ in Figure 5. (A slope of 0.1035 also well explains the effect of velocity on penetration depth for the other three materials.) Residuals, $r_{ij(t)}$, are computed as $d_{ij} - \hat{d}_{ij(t)}$. Table 2 lists the residual values for each material. In Figure 6 these residuals are plotted about the horizontal line, $r_{ij(t)} = 0$.

To determine significance, the $r_{ij(t)}$ are permuted between the materials being compared. Consider, for example, the two DU materials. Their difference is estimated by $b_2 - b_3$ and takes on the value 2.514, the average of the residuals of DU Rc=49 less the average of the residuals of DU Rc=45. The reference distribution for determining significance is constructed by computing $b_2 - b_3$ for each possible combination of the residuals. Figure 7 depicts one such combination where four residuals were reassigned. In that instance $b_2 - b_3 = 1.535$, one of ${}_7C_4 = 35$ reference distribution values computed under the null hypothesis of no difference between the two DU penetrators. Figure 8 displays the reference distribution in the form of a stem-plot. The observed value for $b_2 - b_3$ is circled. There are six distribution values which equal or exceed in absolute value $|b_2 - b_3|$ (denoted by bold type in Figure 8), hence a p-value of 6/35 or 0.171. (Of the two entries in Figure 8 having absolute value of 2.5, one is listed with a superscript to indicate that it would actually appear smaller, in absolute value, than its counterpart if more decimal places were listed.) Table 3 includes the results of each pairwise material comparison. A difference can be claimed between the DU materials

Table 1. Data Matrix for L/D=15

	d_{ij} (mm)	v_{ij} (m/s)	m_{ij1}	m_{ij2}	m_{ij3}
97% tungsten	42.70	1098	1	0	0
	66.80	1304	1	0	0
	78.20	1489	1	0	0
	89.70	1507	1	0	0
DU Rc=49	58.42	1067	0	1	0
	85.34	1314	0	1	0
	101.09	1481	0	1	0
	115.06	1654	0	1	0
DU Rc=45	78.99	1304	0	0	1
	99.06	1482	0	0	1
	116.33	1660	0	0	1
93% tungsten	39.12	1086	0	0	0
	65.02	1297	0	0	0
	83.31	1500	0	0	0
	105.92	1682	0	0	0

Table 2. Residuals Relative to the t^{th} Material for L/D=15

	97% tungsten	DU Rc=49	DU Rc=45	93% tungsten
$r_{ij(t)}$	-0.29	18.65	14.68	-2.63
	2.49	20.00	16.32	1.43
	-5.26	18.46	15.16	-1.29
	4.38	14.52		2.49

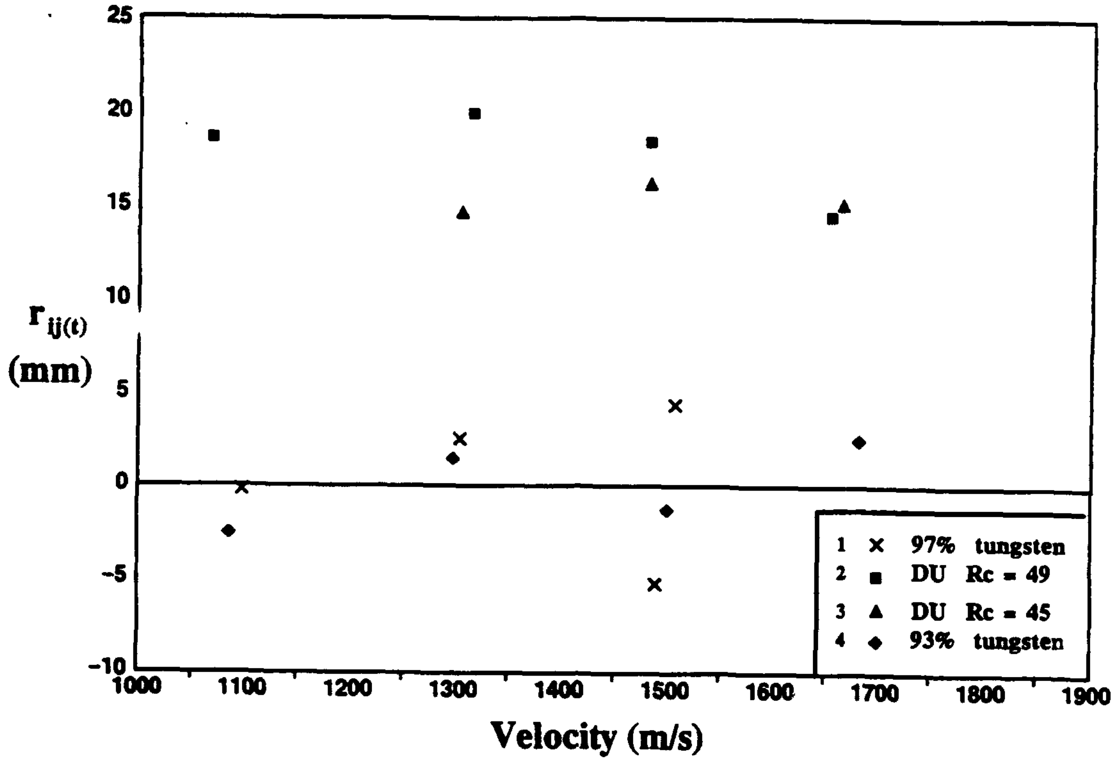

Figure 6. Residuals Relative to the t^{th} Material.

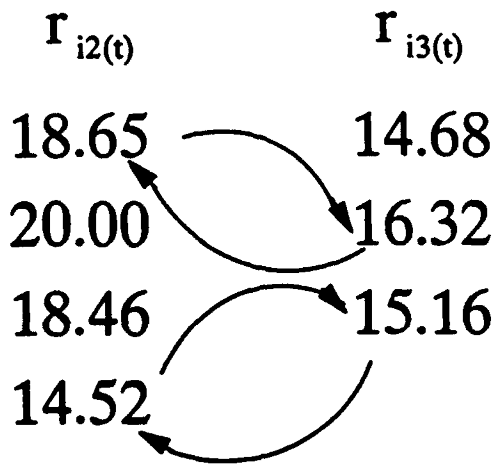

Figure 7. One Possible Reallocation of Residuals
Between Materials 2 and 3.

−3	.9
−2	.6 .5 +
−1	.9 .8 .7 .7 .6 .6 .5 .0
−0	.8 .7 .6 .3 .2
0	.2 .3 .4 .5 .5 .6 .6 .7 .7
1	.2 .3 .3 .4 .5 .6
2	.5 .6 .9
3	.6

Figure 8. Stem-plot Representation of the Distribution of the Test Statistic, $b_2 - b_3$.

Table 3. Significance of the Differences Observed in Example 1

L/D = 15				
		Randomization		
Contrast	Estimate	# unusual	# permutations	p-value
β_1	0.3298	60	70	0.857
β_2	17.9032	2	70	0.029
β_3	15.3890	1	35	0.029
$\beta_1 - \beta_2$	-17.5734	2	70	0.029
$\beta_2 - \beta_3$	2.5142	6	35	0.171
$\beta_1 - \beta_3$	-15.0592	1	35	0.029

with a probability of 0.171 of being wrong. In consideration of the data, all p-values appear reasonable and act to quantify the differences observed.

5.2 Example 2. A second data set is displayed in Figure 9. Three 65-g penetrators were tested, each with L/D = 10. Unlike in the previous example, data were not collected strictly according to the template in Figure 2. They need not be for the randomization test to be valid. Also, the distinction between groups do not appear as great as in Example 1. It is in this situation that an explicit quantification of any differences is most needed because it becomes even less clear how much observed difference is real and how much is attributable to chance variations.

Table 4 lists the results for all pairwise comparisons between materials. The increased sample sizes over the previous example allows for a finer resolution in the number of reference distribution values. There are 12,870 values comprising the reference distribution for b_1, the estimated difference between 97% tungsten and 93% tungsten. The p-value for the randomization test is 0.192, meaning that the probability is 0.192 of observing a value for b_1 at least as unusual as 1.4050. Generally, such a p-value would not be considered significant, suggesting that 97% tungsten and 93% tungsten are performing similarly for L/D = 10 penetrators.

A second contrast $\beta_1 - \beta_2$, signifying the difference between 97% tungsten and DU, is estimated to be -4.5113.* It is not clear from the examination of Figure 9 that this constitutes a real difference in performance. The randomization test, however, yields a p-value of 0.0040 and provides solid justification for the metallurgist's claim that 97% tungsten and DU materials are performing differently. A difference between these materials was observed by Magness (1990).

6. CONCLUSION

For the testing of 1/4-scale kinetic energy penetrators against semi-infinite steel blocks, the technical considerations and the procedures addressing them are long established. It is the intent of this effort to enhance the inferential process within the presiding experimental structure. Presently, once data are collected, inferences principally consist of an engineering judgment as to the meaning of an observed vertical

* No discussion in this report is devoted to controlling the error rate for multiple contrasts. For more detail, see Kirk (1982).

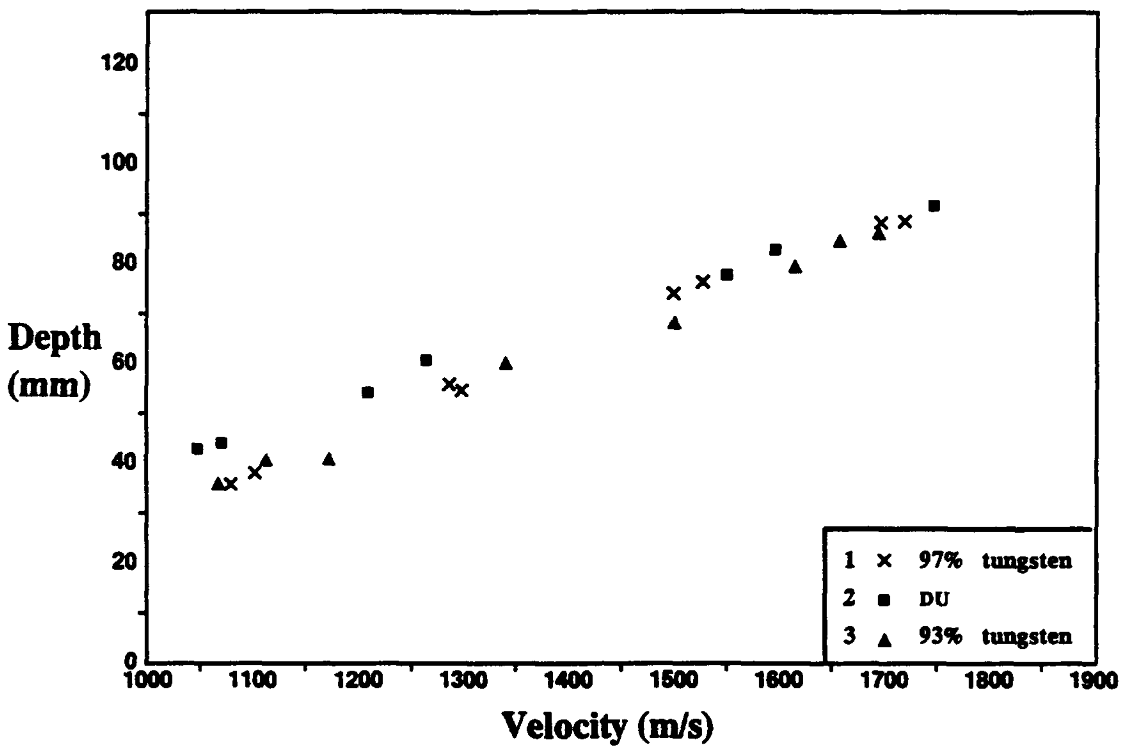

Figure 9. Depth of Penetration for Three Materials, L/D=10.

Table 4. Significance of the Differences Observed in Example 2

L/D = 10				
		Randomization		
Contrast	Estimate	# unusual	# permutations	p-value
β_1	1.4050	2472	12870	0.1921
β_2	5.91630	3	6435	0.0005
$\beta_1 - \beta_2$	-4.5113	26	6435	0.0040

gap between linear functions representing the penetration performance of two materials. The initial motivation for pursuing this problem was the engineer's lament that, occasionally, when his judgment was questioned, he had little recourse but to stand firm on his opinion forged from years of experience. The linear functions themselves are usually established subjectively and are considered parallel over the range of 1100 m/s to 1700 m/s. Such subjectivity does bring into question the consistency of the assessment process. An objective method for fit, such as least squares, is seldom used, and then not in such a way as to incorporate the common slopes assumption. Nor need it be in all instances. Often, the differences are so great as to allow for the approximate fitting of the linear functions with no loss to the outcome, but perhaps equally as often they are not great, occurring when only marginal improvements are made over an historical (control) material.

In summary, the report identifies the experimental situation as being similar to that in which an analysis of covariance model is usually employed and then expresses the linear model in a manner conforming to how practitioners currently view the problem, even to the extent of automatically incorporating the parallel lines assumption. The report then explores some important problems, such as data arising from independent studies, in implementation of the classical method for significance testing and recommends an alternative to surmount these problems in the form of a randomization test. This test is implemented on two sets of real data, and its application in the context of those data is demonstrated.

The approach presented is an attempt at developing a unifying structure within which inferences in this environment can be made both quantifiable and consistent. The recommended procedure combines existing techniques such as least squares with a new application of a randomization test in determining the significance of observed material differences. With this test supporting, practitioners can make definitive statements as to the statistical significance of material differences observed.

7. REFERENCES

Conover, W., and Iman, R. "Analysis of Covariance Using the Rank Transformation." *Biometrics*. 38, 715-724, 1982.

Edgington, E. Randomization Tests (2nd ed.). New York: Marcel Dekker, Inc., 1987.

Farrand, T. Unpublished manuscript, Ballistic Research Laboratory.

Fisher, R. Statistical Methods for Research Workers (4th ed.). London: Oliver and Boyd, 1932.

Hedges, L., and Olkin, I. Statistical Methods for Meta-Analysis. San Diego: Academic Press, Inc., 1985.

Kirk, R. Experimental Design (2nd ed.). Belmont, California: Brooks/Cole, 1982.

Magness, L. Proceedings of the 1990 Army Science Conference, 2, 465-479, 1990.

Neter, J., and Wasserman, W. Applied Linear Statistical Models. Richard D. Irwin, Inc., 1974.

Puri, M., and Sen, P. "Analysis of Covariance Based on General Rank Scores." *The Annals of Mathematical Statistics*, 40, 610-618, 1969.

Quade, D. "Rank Analysis of Covariance." *Journal of the American Statistical Association*, 62, 1187-1200, 1967.

Shirley, E. "A Distribution-free Method for Analysis of Covariance Based on Ranked Data." *Applied Statistics*. 30, 158-162, 1981

Stephenson, W., and Jacobson, D. "A Comparison of Nonparametric Analysis of Covariance Techniques." *Communications in Statistics - Simulation*. 17(2), 451-461, 1988.

Taylor, M., and Bodt, B. "Randomization Procedures Applied to Analysis of Ballistic Data." BRL-TR-3245, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 1990.

Tippet, L. The Method of Statistics. London: Williams & Norgate, 1931.

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GLOSSARY

additive - No terms formed as products of other terms in the model are present.

analysis of covariance - A method involving removing the effect of an explanatory variable on a response, leaving residuals to be analyzed to determine treatment effects.

Central Limit Theorem - Guarantees approximate normality for a sum of responses based on large samples.

combination - Denoted ${}_mC_n$ and meaning the number of groups of n items that can be formed from m items, without regard to order.

contrast - A linear combination of means in which the coefficients sum to zero.

degrees of freedom - Refers to the amount of information yielded by the data. In general, it is desirable to have many degrees of freedom in the denominator of the error term.

factorial design - An efficient method for collecting data characterized by the gathering of data over all treatment level combinations.

fixed effect - A treatment whose influence on the response is to shift the response mean in accordance with the levels of the treatment.

linear model - Refers to a statistical model which is linear with respect to the coefficients to be estimated.

p-value - The probability of being in error when claiming that a difference has been observed (i.e., claiming that the alternative hypothesis is true).

random sampling - A process of selecting n members from a population in such a way that each n-member group has an equally likely chance of selection.

reference distribution - Under the null hypothesis, the possible values that the test statistic might take on and the frequency with which they are taken on.

residual - The departure of the model from the data, measured as the observed data value less the model prediction.

significance - Refers to the magnitude of the p-value.

stem-plot - A graphical display similar to a histogram, where the bars are formed by listing the final digit of all values with the same leading digit(s). The leading digit(s) serve to locate the bar on the axis.

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